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Tobias Herwig

Market-Conform Valuation of Options

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Introduction

1.1 The Area of Research

In this thesis, we will investigate the 'market-conform' pricing of newly issued contingent claims. A contingent claim is a derivative whose value at any settlement date is determined by the value of one or more other underlying assets, e.g., forwards, futures, plain-vanilla or exotic options with European or American-style exercise features. Market-conform pricing means that prices of existing actively traded securities are taken as given, and then the set of equivalent martingale measures that are consistent with the initial prices of the traded securities is derived using no-arbitrage arguments. Sometimes in the literature other expressions are used for 'market-conform' valuation – 'smile-consistent' valuation or 'fair-market' valuation – that describe the same basic idea.

The seminal work by Black and Scholes (1973) (BS) and Merton (1973) mark a breakthrough in the problem of hedging and pricing contingent claims based on no-arbitrage arguments. Harrison and Kreps (1979) provide a firm mathematical foundation for the Black–Scholes–Merton analysis. They show that the absence of arbitrage is equivalent to the existence of an equivalent martingale measure. Under this measure the normalized security price process forms a martingale and so securities can be valued by taking expectations. If the securities market is complete, then the equivalent martingale measure and hence the price of any security are unique. If the market is not complete, a much more realistic assumption in practice, this will no longer hold, so that the investor has to decide how to pick the equivalent martingale measure to be used for pricing.

The approaches in the literature can be divided into two main classes. The first class starts with an assumption about the data-generating process, i.e. about the stochastic process that drives the underlying asset price. The most popular choice for the data-generating process is a geometric Brownian motion, first applied in option pricing theory by Black and Scholes (1973). However, the behavior of implied volatilities derived from inverting the Black–Scholes formula, makes the validity of this model questionable. The empirical evidence provided by, among others, Rubinstein (1994), Jackwerth and Rubinstein (1996), Dumas et al. (1998), or Aït-Sahalia and Lo (1998) shows that implied volatilities vary across different strikes (i.e. they exhibit a smiles or skews pattern) and different times to maturity (term structure), while the BS model does not allow for such variations. These variations can roughly be explained by more sophisticated models, such as stochastic volatility (e.g. Hull and White (1987), Heston (1993), Schöbel and Zhu (1999)), stochastic interest rates (e.g. Merton (1973), Amin and Jarrow (1992)), jump models (e.g. Merton (1976), Bates (1991)), or combinations of the different processes (e.g. Bates (1996), Scott (1997), Bakshi and Chen (1997)). After defining a stochastic process for the underlying, this process has to be rewritten in risk-neutral terms. Then, the parameters of the processes for the underlying asset price and for the volatility and/or jump process are estimated. Most calibration procedures rely on the existence of explicit pricing formulas for the prices of benchmark instruments, since the unknown parameters are found by inverting such pricing formulas. When closed-form expressions exist, the model parameters can often be simply estimated by employing least-squares methods. However, closed-form solutions for prices are not always available or easy-to-compute. In this case, fitting the model to market prices implies searching the parameter space via direct simulation, which is computationally expensive and time-consuming. Finally, after specifying the model parameters of the stochastic processes, the prices of new contingent claims are derived as a function of the parameters of these processes and the price of the underlying asset.

Unfortunately, these models often do not fit observed market prices accurately (e.g. Das and Sundaram (1999), Belledin and Schlag (1999)). Therefore, they should be used carefully in practice, especially to price and hedge exotic options. This is due to the fact that in order to improve the hedging performance, exotic and standard options need to be valued consistently, since exotic options are often hedged with portfolios of European options. These problems are discussed in the literature on 'market-conform' or 'smile-consistent' no-arbitrage models, the second class of no-arbitrage approaches.

Market-conform models reverse the approach followed in the conventional stochastic volatility or jump models. The prices of actively traded European options are taken as given, and they are used to infer information about the underlying price process. The implementation of market-conform models for pricing and hedging purposes is mainly done in a discrete time framework. The tools used are either implied binomial/trinomial trees, implicit finite difference schemes, or weighted Monte Carlo simulations.

The most popular 'market-conform' approaches are the so-called implied tree models, which are extending the seminal binomial model of Cox et al. (1979). In the standard Cox et al. (1979) tree, the size of the up and down move of the underlying and the respective transition probability of such moves are constant, since they depend on the volatility, which is assumed to be constant. This is no longer the case for implied trees. Implied binomial (or trinomial) trees are built from the known prices of European options. In order to build a consistent risk-neutral price process of the underlying, these exchange-traded options are used to infer information about the data-generating process. They are called 'implied trees', because they are consistent with or implied by the volatility structure and can be viewed as a discretization of generalized one-dimensional diffusions in which the volatility parameter is allowed to be a function of both time and asset price.

We propose a new method to construct arbitrage-free implied binomial trees based on the approach by Brown and Toft (1999). As the output of our procedure we get an arbitrage-free, risk-neutral implied binomial tree, which is consistent with the term structure of implied volatilities and also with the implied volatility smile. The implied risk-neutral probability distributions (IRNPDs) for later maturity dates are an endogenous result of the model and take the IRNPDs of the prior maturity dates into account. Our method can also be used to construct arbitrage-free, risk-neutral implied multinomial trees. This multinomial setting can be used to calibrate models with more than one state variable, e.g. the underlying price process and stochastic volatility. Since the approaches suggested by Rubinstein (1994), Dupire (1994), Derman and Kani (1994), Derman et al. (1996), Jackwerth (1997), Barle and Cakici (1998), and Brown and Toft (1999) are closely related to our new technique, we briefly describe the differences and, in particular, the drawbacks of these models. For more detailed surveys see Jackwerth (1999) or Skiadopoulos (2001).

The key idea of the approach suggested by Rubinstein (1994) is the estimation of the IRNPD at the terminal date of the tree. This IRNPD is close to a prior guess subject to some constraints. There are two major drawbacks of this method. First, the implied binomial tree fits the strike dimension of the volatility smile only at one single maturity date and neglects information for traded options with shorter or longer maturities. This leads directly to the second problem, namely the fact that implied binomial trees constructed for two different maturity dates are not necessarily consistent for overlapping time-periods. To overcome these problems, Jackwerth (1997) develops a generalized implied binomial tree by introducing a piecewise-linear weight function and by using nodal probabilities instead of path probabilities. This generalization allows for the incorporation of all the information and fits the complete volatility surface. However, the calibration requires a non-linear optimization approach to fit the tree and can become computationally expensive. Both aforementioned approaches estimate the terminal IRNPD and work backwards in time. Furthermore, the implied binomial trees are arbitrage-free by construction. Another way to construct an implied binomial tree was suggested by Brown and Toft (1999). They use a three-step procedure and get an arbitrage-free, semi-recombining implied binomial tree, which is consistent with the implied volatility surface. However, this method does not use all the available information optimally, since the IRNPD for each maturity date is estimated separately. Moreover, in some cases, the optimization problem can not be solved, since the constraints of the optimization problem can not be satisfied.

Derman and Kani (1994) construct an implied tree assuming that any option value can be interpolated or extrapolated from the prices of actively traded options. Therefore, the resulting implied tree fits the volatility smile in strike and time dimension. Unfortunately, negative transition probabilities can occur and must be replaced by values between zero and one. This may lead to numerical instability of the tree, especially for a large number of time steps. Barle and Cakici (1998) extend the approach by Derman and Kani (1994) to reduce the numerical problems and increase the stability of the algorithm. However, even this approach does not guarantee positive transition probabilities in all nodes.

The main characteristic of implied trinomial trees is that the complete state space for the tree is fixed in advance and only the transition probabilities must be calculated. To construct the implied trinomial tree, missing prices must also be calculated by interpolation or extrapolation of the implied volatility smile. Therefore, the performances of the models depend on the respective interpolation or extrapolation