

R. G. Ballas

Piezoelectric Multilayer Beam Bending Actuators

Static and Dynamic
Behavior and Aspects
of Sensor Integration

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Static and Dynamic Behavior
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With 143 Figures and 17 Tables

 Springer

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Dedicated to my parents Helga and Egon Ballas

Preface

In recent years, solid state devices utilizing piezoelectric effects have drawn much attention both scientifically and technologically because piezoelectric actuators, sensors and transducers have been widely used in many electromechanical applications including active and passive vibration damping, ultrasonic motors, ultrasonic biomedical imaging, loudspeakers, accelerometers, resonators, micropositioner, acoustic sensor, etc. Many new devices and applications are explored intensively, and many new technological developments based on these devices are emerging. In the field of piezoelectric actuators, one of the major parts is represented by beam bending actuators, which are in the focus of the book.

The broad field of industrial applications for piezoelectric beam bending actuators as actuating and high precision positioning systems necessitates an adjustment of their performance for the particular task. Regarding piezoelectric beam benders in multilayer technology, a deeper inside look and understanding of their mechanical, elastic and electromechanical characteristics is required. This deeper understanding discloses the possibility of making statements about the beam bender's static and dynamic behavior and its influencing by appropriate measures within the scope of structural mechanics.

The book is divided into five parts. Part 1 gives an introduction to the topic and highlights the individual aspects presenting the focus of the book. Part 2 consists of chapters 2 to 7. In these chapters, theoretical approaches concerning the description of the static and dynamic behavior of piezoelectric multilayer beam bending actuators are discussed in detail. The achieved results are represented in closed form analysis. Part 3 of the book dwells on the experimental characterization of piezoelectric bending actuators represented in the chapters 8 and 9. Different static and dynamic characteristics of a realized bending actuator are determined experimentally and compared with analytical calculations based on the theoretical considerations made before. Part 4 of the book consists of chapters 10 to 12 addressing the sensor inte-

gration into a piezoelectric bending actuator. By means of both, a capacitive and inductive displacement sensor, the tip deflection of a bending actuator is measured. The aspect of sensor integration allows for new applications where high-accuracy positioning is required. Part 5 of the book contains a detailed appendix with important mathematical and physical aspects, that would go beyond the scope of the book's mainmatter. The book is suitable as a reference for graduate students, engineers and scientists working in industry and academia. An introductory course on mechanics of materials, elasticity, theoretical mechanics, electronics and network theory should prove to be helpful but not necessary because the basics are included in the relevant chapters.

I gratefully acknowledge the support and encouragement of Darmstadt University of Technology, Institute for Electromechanical Design (EMK) in carrying out the research and the writing of this book. I am highly indebted to a number of people who significantly contributed to the manuscript. Helmut F. Schlaak, head of the Institute for Electromechanical Design, always supported my research with his expert advice. Roland Werthschützky gave me deep insights into the interesting field of electromechanical systems and their circuit representations. They form a main part of the book. I acknowledge Heinz Lehr for the productive discussions concerning the theoretical mechanics being necessary to get deeper insights into mechanical and electromechanical systems. My special thanks go to Ralf Greve, who sparked my interest in theoretical physics, particularly with regard to the theoretical mechanics, that supplied the fundamentals for the dynamic considerations of piezoelectric bending actuators. It is a pleasure to acknowledge the help of the following colleagues in proof reading of the preliminary manuscript: Dirk Eicher, Peter Lotz, Jaqueline Rausch, Marc Matysek, Stephanie Klages, Christian Wohlgemuth, Andreas Röse, Benedikt Schemmer, Ingmar Stöhr and Lutz Rafflenbeul. I gratefully acknowledge Felix Greiner, Alexander Mössinger, Alexander Kolb, Reinhard Werner and Sebastian Nolte who supported my research by their helpful activities in the laboratory and their suggestions. Further, my thanks go to the Eva Hestermann-Beyerle and Monika Lempe of Springer-Verlag, who offered an excellent cooperation and continous support. Special thanks go to my parents Helga and Egon Ballas for their love and patience while I was writing this book.

Darmstadt, November 2006

Rüdiger G. Ballas

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List of Symbols

Symbol	Quantity	Unit
A	surface	m^2
\mathbf{A}	two-port network	
A_i	surface of the i th layer	m^2
A_{tot}	total gain factor	1
A_ν	flux cross-section in section ν	m^2
A_0, A_1, A_2	gain factors	1
$\underline{A}_1, \underline{A}_2$	integration constants	A
a	coil radius	m
\underline{a}	acceleration	m s^{-2}
a_j	coefficient (ansatz function)	
a, c	crystal axes	
a_1, b_1	coefficients	
\mathbf{B}	vector of magnetic induction	T
B_0	transfer coefficient	V F^{-1}
B_ν	magnetic induction in section ν	T
B_{n1}, B_{n2}	normal components of the mag. induction	T
\underline{B}_0	total transfer function	1
$\underline{B}_1, \underline{B}_2, \underline{C}_1$	integration constants	A
$\underline{B}_3, \underline{B}_A, \underline{B}_g$	transfer functions	
b	width (dielectric)	m
C	flexural rigidity	N m^2
	capacitance	F
C_b	translatory fixed capacitance	F
C_{tot}	total capacitance	F
C_i	capacitance of layer i	F
C_{ref}	reference capacitance	F
C_x	capacitance to be measured	F
C'_F, C'_{Piezo}	capacitance per unit length	F m^{-1}
C_0, C_T, C_{x0}	nominal capacitance	F

Symbol	Quantity	Unit
c_{pq}^E	modulus of elasticity at const. elec. field	N m^{-2}
c_{ijkl}, c_{pq}, E	modulus of elasticity	N m^{-2}
D	Rayleigh's dissipation function	J s^{-1}
D	finger width (interdigital structur)	m
\mathbf{D}	vector of electric displacement	C m^{-2}
D_j, \underline{D}_j	component of elec. displacement vector	C m^{-2}
$D_{j,i}, \underline{D}_{j,i}$	component of elec. displacement (<i>i</i> th layer)	C m^{-2}
d_a	planar coil distance	m
d_b	effective diameter (planar coils)	m
d_{Fe}	layer thickness (iron core)	m
d_{air}	air gap	m
d_{Ni}	layer thickness (thermal adjustment layer)	m
d_{jq}	piezoelectric coefficient	m V^{-1}
$d_{jq,i}$	piezoelectric coefficient (<i>i</i> th layer)	m V^{-1}
$d_F, d_{F,1}$	interdigital periode	m
E	vector of electric field	V m^{-1}
E_c	coercive field strength	V m^{-1}
E_j, \underline{E}_j	components of electric field vector	V m^{-1}
$E_{j,i}, \underline{E}_{j,i}$	comp. of elec. field vector (<i>i</i> th layer)	V m^{-1}
e_{iq}	piezoelectric modulus	C m^{-2}
$\mathbf{e}_r, \mathbf{e}_\varphi, \mathbf{e}_z$	unit vectors (cylindrical coordinates)	
$\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$	unit vectors (Cartesian coordinates)	
F	force, extensive quantity	N
F	force vector	N
F_b	actuator force	N
F_G	total error	
F_j	components of force vector	N
F_N	axial force	N
F_Q	lateral force	N
F_{tot}	total force	N
F_{Hys}	hysteresis error	
$\mathbf{F}_i^{(ext)}$	vector of ext. force on mass point <i>i</i>	N
$\mathbf{F}_i^{(r)}$	vector of frictional force on mass point <i>i</i>	N
$\mathbf{F}_i^{(z)}$	vector of constraining force on mass point <i>i</i>	N
F_j	components of force vector	N
\mathbf{F}_{ij}	interaction force	N
$F_{j,i}$	axial force in the <i>i</i> th layer	N
F_{Lin}	linearity error	
\underline{F}_r	frictional force	N
\underline{F}_s	force generated by a bender's tip	N
F_{Piezo}	piezoelectric force	N
$\mathbf{F}_i, \mathbf{F}_i^{(a)}$	force applied on mass point <i>i</i>	N
$\underline{F}, \underline{F}_1, \underline{F}_2$	force (translatory flow quantity)	N
f_0	resonant frequency	Hz
f_A	force per unit area	N m^{-2}
f_e	effective frequency	Hz

Symbol	Quantity	Unit
f_s	switching frequency	Hz
f_T	carrier frequency	Hz
f_u	lower frequency	Hz
f_{\max}	maximum frequency	Hz
f_{meas}	measured frequency	Hz
f_{mod}	modulation frequency	Hz
f_{Piezo}	piezoelectric force per voltage	N V ⁻¹
f, \underline{f}	load per unit length	N m ⁻¹
f_1, f_2	eigenfrequencies	Hz
$f_{\text{Mess1}}, f_{\text{Mess2}}$	measured frequencies	Hz
$f_{\text{osc1}}, f_{\text{osc2}}$	resonant frequencies	Hz
$f_{\text{out}}, \underline{f}_{\text{out}}, f_{\text{schmitt}}$	output frequencies (ASIC)	Hz
f_x, f_y, f_z	volume forces	N m ⁻³
G_x	parallel conductance of capacitance	S
g	gravitational acceleration	m s ⁻²
H	vector of magnetic field	A m ⁻¹
H, <u>H</u>	admittance matrix	
H_e	electrical enthalpy	J m ⁻³
H_ν	magnetic field in section ν	A m ⁻¹
$H_{e,i}$	electrical enthalpy of the i th layer	J m ⁻³
$\underline{H}_{\text{coil}}, \underline{H}'_{\text{coil}}$	magnetic field in the circular loop plane	A m ⁻¹
H_{t1}, H_{t2}	tangential components of magnetic field	A m ⁻¹
$\underline{H}(t), \underline{H}'(t)$	complex time function of magnetic field	A m ⁻¹
h	layer thickness (inductive sensor layer)	m
h_i	height of i th layer	m
h	layer thickness (piezoelectric sensor layer)	m
$h_{ij}, \underline{h}_{ij}$	matrix elements of the admittance matrix	
$h_{i,o}$	upper distance of i th layer from neutral axis	m
$h_{i,u}$	lower distance of i th layer from neutral axis	m
h_0, h_1	layer thickness (dielectrics)	m
$\underline{h}, \underline{h}_1, \underline{h}_2$	mechanical admittance	s kg ⁻¹
I_{coil}	coil current	A
I_{yy}	geometrical moment of inertia	m ⁴
$\underline{I}(t)$	complex time function of current	A
$\underline{I}, \underline{I}_1, \underline{I}_s$	complex current amplitudes	A
\dot{i}	current (electric flow quantity)	A
i_s	eddy current	A
\dot{i}_w	transducer current	A
i_1, i_2	alternating current	A
i, j	summation indices	
J	vector of current density	A m ⁻²
J_0, J_1	Besselfunctions of first kind (order 0 and 1)	
K_{NI}	motor parameter	N A ⁻¹
k	wavenumber	m ⁻¹
	amount of ansatz functions	
	k -factor	1
k_m	wavenumber of the m th eigenmode	m ⁻¹

Symbol	Quantity	Unit
$k(x)$	coupling factor (transformer)	1
$k_{31,i}^2$	electromechanical coupling factor	1
L	Lagrangian function	J
	inductance of magnetic circuit	H
L_0	nominal inductance	H
	overlap (interdigital structure)	m
L_e	effective inductance	H
L_s	inductance (inductive sensor layer)	H
L_1, L_2	inductances	H
l	length of bending actuator	m
	length of coil	m
	length of dielectric	m
l_ν	average length of field line	m
M	bending moment, extensive quantity	N m
$M(x)$	mutual inductance	H
\mathbf{M}	coupling matrix	
M_{tot}	total moment	N m
\underline{M}_w	transducer moment	N m
m	vibrational mode	
	symbol for a symmetry plane	
m_A	moment per unit area	N m ⁻¹
m_i	point mass	kg
m_{ij}	matrix element of the coupling matrix	
M_{Piezo}	piezoelectric moment	N m
m_{Piezo}	piezoelectric moment per voltage	N m V ⁻¹
$\underline{M}, \underline{M}_1, \underline{M}_2$	moment (rotatory flow quantity)	N m
N_e	amount of carrier	1
N_F	amount of interdigital periods	1
n	amount of layers	1
n_0	translatory reference compliance	m N ⁻¹
n_R	torsional compliance	N ⁻¹ m ⁻¹
n'_R	torsional compliance per unit length	N ⁻¹ m ⁻²
n_{Rk}	torsional short-circuit compliance	N ⁻¹ m ⁻¹
N, N_1, N_2	turns	1
\mathbf{P}	vector of electrical polarization	C m ⁻²
\mathbf{P}_r	vector of remanent polarization	C m ⁻²
\mathbf{P}_s	vector of spontaneous polarization	C m ⁻²
p	pressure load, extensive quantity	bar
	amount of holonomic constraints	
\mathbf{p}_i	impuls of mass point i	N s
Q	charge	C
	thermal energy density	J m ⁻³
	Q -factor	1
\underline{Q}	charge (complex)	C
Q_j	generalized force	
Q_m	Q -factor of the m th vibrational mode	1
Q_{tot}	total charge	C

Symbol	Quantity	Unit
\tilde{Q}	thermal energy	J
$Q_j^{(ext)}$	external generalized force	
$Q_j^{(k)}$	conservative generalized force	
$Q_j^{(r)}$	generalized frictional force	
Q_i, \underline{Q}_i	charge in the i th layer	C
q	amount of periods	1
	transformation ratio	m
q_j	generalized coordinate	
\dot{q}_j	generalized velocity	
R	resistance	Ω
R_e	effective resistance	Ω
R_m	magnetic resistance	A Wb ⁻¹
R_s	resistance (inductive sensor layer)	Ω
R_{int}	resistance (integrator)	Ω
r	radius	m
	coefficient of friction	N s m ⁻¹
\mathbf{r}	position vector	m
\mathbf{r}_d	difference vector	m
r_a	coefficient of friction per unit length	N s m ⁻²
r, φ, z	cylindrical coordinates	
R_2, R_{ref}	resistance of potentiometer	Ω
R_{air}, R_{Ni}, R_{Fe}	magnetic resistances	A Wb ⁻¹
R_1, R_2, R_3, R_4	resistances (Wheatstone bridge)	Ω
S	mechanical strain	1
	degrees of freedom	
	integral of action	
\bar{S}	average mechanical strain	1
\underline{S}	mechanical strain (complex)	1
S_r	remanent strain	1
S_{ij}, S_p	strain tensor	1
$\tilde{S}, \tilde{C}, \tilde{s}, \tilde{c}$	Rayleigh functions	
s_n	standardized frequency	1
s_{pq}^E	elastic compliances at const. elec. field	m ² N ⁻¹
$s_{pq,i}$	elastic compliance of the i th layer	m ² N ⁻¹
$s_{pq,i}^E$	elast. compl. of i th layer at const. el. field	m ² N ⁻¹
s_{ijkl}, s_{pq}	elastic compliances	m ² N ⁻¹
T	kinetic energy	J
	oscillation period	s
	temperature	K
\mathbf{T}	vector of mechanical stress	N m ⁻²
T_0	room temperature	K
T_C	Curie temperatur	K
T_{ij}, T_q	stress tensor	N m ⁻²
$T_{q,i}, \underline{T}_{q,i}$	mechanical stress in the i th layer	N m ⁻²
t	time	s
U	voltage, extensive quantity	V
	internal energy density	J m ⁻³

Symbol	Quantity	Unit
U	bridge voltage	V
U_s	driving voltage	V
\tilde{U}	internal energy	J
\underline{U}_0	voltage (circuit representation of a transformer)	V
\underline{U}_e	driving voltage	V
\hat{U}_e	peak value	V
U_{DC}	DC offset	V
U_{agl}	rectified voltage	V
U_{force}	controller signal	V
U_{pos}	position proportional voltage	V
U_{max}	maximum driving voltage	V
U_{Range}	voltage range	V
U_{set}	set value	V
U_{0x}	DC offset of sensor electronics component x	V
U_0, U_{offset}	offset voltage	V
$\underline{U}_a, \underline{U}_{a1}, \underline{U}_{a2}$	output voltages	V
\mathbf{u}	vector of total displacement	m
u	eigenvalues (continuous spectrum)	m^{-1}
u_0	displacement in x -direction	m
	voltage amplitude	V
u_n	eigenvalues (discret spectrum)	m^{-1}
$u_{f/U}$	output voltage (f/U converter)	V
\underline{u}	voltage (electrical difference quantity)	V
\underline{u}_N	nominal voltage	V
\underline{u}_{mod}	modulation voltage	V
$\underline{u}_{mod,ss}$	peak-to-peak voltage (modulation voltage)	V
\underline{u}_{net}	output voltage network analyzer	V
\underline{u}_{Piezo}	output voltage amplifier stage	V
\underline{u}_{tri}	output voltage laser triangulator	V
u_1, u_2	voltages (transformer)	V
u, v, w	displacement	m
V	volume	m^3
	volume displacement	m^3
	potential energy, potential function	J
V_0, V_1	volume (dielectrics)	m^3
\mathbf{v}_i	vector of velocity of mass point i	m s^{-1}
\underline{v}_s	translatory velocity	m s^{-1}
$\underline{v}, \underline{v}_i$	velocity	m s^{-1}
W_a	final value work	J
W_e	electrical energy	J
W_m	mechanical work	J
W_F	mechanical work (force)	J
W_M	mechanical work (moment)	J
W_p	mechanical work (pressure load)	J
W_r	frictional work	J
W_{tot}	total energy	J
$W_{tot,i}$	total energy in the i th layer	J

Symbol	Quantity	Unit
w_e	electrical energy density	J m^{-3}
w_{tot}	total energy density	J m^{-3}
w_i	width of i th layer	m
w_L	gap (interdigital structure)	m
w_m	energy density of elastic deformation	J m^{-3}
$w_{tot,i}$	total energy density in the i th layer	J m^{-3}
$X(x)$	eigenfunction, eigenmode	
$X_m(x)$	eigenfunction m , eigenmode m	
X_k, Z	coordinate axes, crystal axes	
\underline{x}_a	output quantity	
\underline{x}_e	input quantity	
x_0	point of affecting load	m
	starting position	m
$\mathbf{x}_p, \mathbf{x}_{p'}$	position vectors	m
x, \underline{x}	distance	m
x, y, z	Cartesian coordinates	m
x_i, y_i, z_i	components of i th mass point	m
\mathbf{Y}	gyrator constant	C^{-1}
\mathbf{Y}'	integrated gyrator constant	m C^{-1}
\underline{Z}	impedance	Ω
z	distance from the neutral axis	m
\bar{z}	neutral axis position	m
\underline{z}_1	mechanical impedance	kg s^{-1}
z_{ij}	elements of the dynamic admittance matrix	
α	bending angle	rad
$\beta_m(\omega)$	admittance of the m th vibrational mode	s kg^{-1}
ΔA_i	surface segment (layer i)	m^2
ΔC_b	translatory fixed capacitance	F
$\Delta C, \Delta C_x$	capacitance change	F
Δf	frequency change	Hz
Δf_T	carrier frequency change	Hz
ΔF	resulting force (actuator segment)	N
$\Delta \underline{i}$	current (actuator segment)	A
$\Delta \underline{i}_w$	transducer current (actuator segment)	A
$\Delta L_1, \Delta L_{Range}$	inductance change	H
Δl	length change	m
Δm	mass (actuator segment)	kg
Δn_R	torsional compliance (actuator segment)	$\text{N}^{-1} \text{m}^{-1}$
Δn_{Rk}	torsional short-circuit compliance	$\text{N}^{-1} \text{m}^{-1}$
$\Delta \underline{Q}_i$	charge in the i th layer (actuator segment)	C
ΔR	resistance change	Ω
Δr	coefficient of friction (actuator segment)	N s m^{-1}
Δt	time difference	s
ΔU	voltage step	V
$\Delta \underline{U}$	induced voltage	V
$\Delta U_{agl}, \Delta \hat{U}_e$	voltage changes	V
Δx	length (actuator segment)	m

Symbol	Quantity	Unit
$\Delta\Phi$	change of magnetic flux	Wb
$\Delta\varphi, \Delta\phi$	angle difference	rad
$\Delta\omega$	frequency change	s ⁻¹
$\Delta\Omega$	angular velocity (actuator segment)	s ⁻¹
δ	variational operator	
	equivalent conductive layer thickness	m
$\delta(x)$	Dirac delta function	m ⁻¹
$\delta\mathbf{x}, \delta\mathbf{x}_i$	virtual displacements	m
$\delta\phi$	virtual torsion	rad
ε	permittivity	F m ⁻¹
ε^0	strain of the neutral axis	1
ε_r	relative permittivity	1
ε_{eff}	effective permittivity	F m ⁻¹
ε_{jk}	permittivity tensor	F m ⁻¹
ε_{jk}^S	permittivity at constant strain	F m ⁻¹
ε_{jk}^T	permittivity at constant stress	F m ⁻¹
$\varepsilon_{jk,i}^S$	permittivity of <i>i</i> th layer at constant strain	F m ⁻¹
$\varepsilon_{jk,i}^T$	permittivity of <i>i</i> th layer at constant stress	F m ⁻¹
ζ_m	attenuation constant of <i>m</i> th eigenmode	1
$\eta(x)$	test function	
η_m	frequency ratio	1
Θ	temperature	K, °C
κ	conductivity	S m ⁻¹
κ^0	bend of the neutral axis	m ⁻¹
Λ	logarithmic decrement	1
λ_m	eigenvalue of the <i>m</i> th eigenmode	m ⁻⁴
λ_{Piezo}	capacitance per unit length	F m ⁻¹
μ	mass per unit length	kg m ⁻¹
	permeability	N A ⁻²
μ_e	carrier mobility	m ² Wb ⁻¹
μ_N	physical nominal measured quantity	
μ_r	relative permeability	1
μ_ν	permeability in section ν	N A ⁻²
$\xi, \underline{\xi}$	deflection	m
ξ_j	ansatz function	m
ξ_0	displacement in <i>z</i> -direction	m
ξ_{max}	maximum deflection	m
ξ_{Range}	absolute deflection range	m
Π	total potential energy	J
ρ	charge density	C m ⁻³
ρ_i	mass density of the <i>i</i> th layer	kg m ⁻³
$\tilde{\Sigma}$	entropy	J K ⁻¹
σ_A	surface density	C m ⁻³
τ_m^d	period (subcritically damped oscillation)	s
Φ_{coil}	magnetic flux (coil plane)	Wb
Φ, Φ_1, Φ_2	magnetic flux	Wb
ϕ	phase angle	rad

Symbol	Quantity	Unit
$\phi_m^h(t)$	homogeneous solution (free damped vibration)	m
$\phi_m^p(t)$	particular solution (forced vibration)	m
ϕ_0, ϕ_p	maximum amplitude	m
φ	electric potential	V
	angle	rad
φ_{out}	phase of the output signal	rad
Ψ	phase angle	rad
ω	frequency	s ⁻¹
Ω	excitation frequency	s ⁻¹
$\Omega, \underline{\Omega}_i$	angular velocity	s ⁻¹
ω_0	frequency of the fundamental mode	s ⁻¹
	cutoff frequency	s ⁻¹
ω_m	frequency of the m th vibrational mode	s ⁻¹
ω_m^d	frequency (free damped flexural vibration)	s ⁻¹
$\underline{\Omega}_w$	angular velocity (rotatory difference quantity)	s ⁻¹

Focus of the Book

Introduction

The piezoelectric effect, which was discovered for the first time by the brothers Pierre and Jacques Curie, combines electrical with mechanical quantities and vice versa. If piezoelectric materials (e.g. quartz, turmalin) are subjected to electrical signals along certain crystal orientations, deformations along well-defined crystal orientations appear. Contrary, a mechanical deformation results in a generation of polarization charges. However, the piezoelectric effect can not be assigned to monocrystalline materials only. Conditional on the development of polycrystalline materials with piezoelectric properties, the piezoelectrics achieved an enormous technical relevance as functional materials. Nowadays, polycrystalline ceramics like *barium titanate* (BaTiO_3) and *lead zirconate titanate* (PZT) belong to the most commonly used piezoelectric materials, in particular due to the low manufacturing costs and the almost arbitrary shaping possibilities compared to single crystalline piezoelectrics. Furthermore, they have outstanding piezoelectric and dielectric properties, which makes them particularly indispensable for the field of actuators [1].

1.1 Application Areas of Piezoelectric Actuators

The field of application of piezoelectric actuators extends over mass production applications like sound transmitters, ultrasonic power transducers and sensors, bending actuators for textile machines, ink print heads, beam benders in valves, in braille displays, in optical systems and newly as monolithic multilayer actuators for automotive injection systems. Reasons are their compact and space-saving constructions, high actuating precision, extremely short response times, absence of friction, vacuum and clean room capability and the possibility of operation at cryogenic temperatures [2-4].

The major part of piezoelectric actuators is represented by stack and beam bending actuators. Stack actuators, which are based on the longitudinal piezoelectric effect, consist of several ceramic layers with changing polarity. Inbe-

tween, contact electrodes for actuator driving are provided. With this type of actuator very high forces with however small elongations in the lower μm -range and very high driving voltages in the kV-range can be realized. Stack actuators are used for different applications, e.g. in the automotive industries as injecting valve drives and in the optics as high precision drives.

Beam bending actuators however are based on the transverse piezoelectric effect and are applied where large deflections are needed. The small transversal length variations of the active piezoelectric layers effected by the structure of actuator and the external clamping, result in an internal piezoelectric moment, which is the cause for the bending deformation. Thus, large deflections in the range of several hundred microns are realizable. Due to the larger compliances of beam bending actuators compared to those of stack actuators lower driving voltages are necessary, that lie in the range of approx. 24 - 200 V [5].

The *monomorph* (one active piezoelectric and one passive flexible layer), the *bimorph* (two active piezoelectric layers) and the *trimorph* (one passive flexible layer symmetrically surrounded by two active piezoelectric layers) belong to the most well known representatives of piezoelectric beam bending actuators. Newer developments have resulted in the realization of monomorph beam structures in *multilayer technology*, i.e. the actuator consists of several passive flexible and active piezoelectric layers. The multilayer technology implies the advantage to work with even lower driving voltages extending significantly the field of industrial applications, piezoelectric beam bending actuators can be used for.

1.2 Motivation and Aim of the Book

The consideration of piezoelectric beam bending actuators as a system consisting of a multiplicity of layers (multilayer beam benders) with different mechanical and electromechanical characteristics form the first main emphasis of the present book.

The broad field of industrial applications for piezoelectric beam bending actuators as actuating and high precision positioning systems necessitate an adjustment of their performance for the particular task. Regarding piezoelectric multilayer beam benders, a deeper look and understanding of their mechanical, elastic and electromechanical characteristics is required. This deeper understanding discloses the possibility of making statements about the beam bender's static and dynamic behavior and affecting this behavior by appropriate measures within the scope of structural mechanics.

An essential aim of this book consists in deriving the static and dynamic behavior for any kind of clamped-free piezoelectric beam bending actuator

consisting of n layers in closed form analysis for any point over the entire beam length. The origin of these considerations are the stress and deformation states of a bending bar. In particular, the main focus is turned to the influences on the resulting static and dynamic bending behavior caused by external affecting mechanical quantities, like forces, moments and pressures but also internal piezoelectric moments caused by an external driving voltage. The modeling is to consider any layer sequence consisting of passive elastic and active piezoelectric layers with respect to their geometrical, elastic and electromechanical characteristics. Within the scope of the dynamic characterization, the emphasis is particularly laid on the spatial and temporal resolution of the dynamic vibration behavior.

Piezoelectric bending actuators are really a prime example for an electromechanical system. Generally, electromechanical systems consist of interacting electrical, mechanical and acoustical subsystems, that can be described by circuit representations within the scope of the network theory. This kind of representation provides deep insights into the interconnecting structure between electrical and mechanical subsystems and extends the systematic understanding in the field of actuators as well as in the field of sensor technology. Starting from the attained knowledge concerning the dynamic behavior of piezoelectric multilayer beam bending actuators, the systematic development of their equivalent network representation is effected in a next step within the scope of the network theory.

In order to verify the developed structural dynamic formulations as well as the equivalent circuit representation within the context of the network theory, real measurement results of different kind at a monomorph beam bending actuator in multilayer technology will be carried out with the help of an especially realized measurement setup for piezoelectric bending actuators. In a next step, analytical computations based on the developed formalisms will be compared to the achieved measurement results.

The second main focus of the this book concentrates on the development and realization of a smart sensor-actuator-system. Due to the high electrical driving voltages inside of the piezoceramic layers, microphysical domain processes proceed, resulting in hysteresis, creep and drift effects on a macroscopic level. These effects turn out to be a significant disadvantage, thus piezoelectric bending actuators can only be employed for high accuracy positioning applications under certain circumstances.

For applications where precise switching and exact position control are needed, action has to be taken for the compensation of the inherent piezoelectric creep and hysteresis effects as well as the external subjecting quantities, such as varying mechanical loads and vibrations. With the help of an integrated sensor and additional sensor electronics the disturbing effects can be detected. The

implementation of such a sensor-actuator-system into a closed loop control allows for compensation of disturbing effects, whereby high efficient piezoceramics with high elongation and force characteristics can be used for high-accuracy positioning.

A part of the content of the present book is embedded in the federal german research project EPIETEC (*Effektive piezokeramische Multilayer-Technologie für integrierte Niedervolt-Piezobieger*), which has been financially supported by the German Federal Ministry of Education and Research (BMBF) by contract No. 03N1076A. Within this project, a new pneumatic micro valve has been realized. The heart of this micro valve is a smart piezoelectric low-voltage multilayer beam bending actuator with an integrated sensor for tip deflection measurements.

1.3 State of the Scientific Research

In the scientific literature, a huge number of publications concerning the static and dynamic behavior of piezoelectric beam bending actuators and their electromechanical network representation can be found. Thereby monomorphs, bimorphs, trimorphs and multilayer beam bending structures are in the focus of attention. Mentioning all important works in this area in its completeness would go beyond the scope of the present book. Instead, the most important publications serving as a base for this book are presented briefly.

Statics of Piezoelectric Beam Bending Actuators

The electromechanical coupling mechanisms of monomorphs and bimorphs are discussed on the base of the deformation state describing equations by *Wang* and *Cross* [6]. In a further step, the investigation of the nonlinear behavior of piezoelectric bimorph structures under exposure to high electric fields takes place both, in analytic and experimental form [7]. *Cunningham*, *Jenkins* and *Bakush* pursue in their work the optimization of the layer thickness of a piezoelectric actuator in order to reach maximum deflection of a coupled flexible, non-piezoelectric layer [8]. *Küppers* presents the realization of a miniaturized piezoelectric bending actuator in a monomorph design. In order to optimize the actuator characteristics, analytical computations regarding the deflection of a two layer system are carried out [9]. The modeling of asymmetrical piezoelectric bimorph structures is discussed in the work by *Brissaud*, *Ledren* and *Gonnard*. Considering the neutral axis position, the static behavior regarding the expected bending moment is determined [10]. An analytical description of the free tip deflection of a piezoelectric bimorph by means of matrix calculus is presented by *Huang*, *Lin* and *Tan* [11].

Low and *Guo* as well as *Wang* and *Cross* extend the universal deformation state equations to trimorph bending structures [12, 13]. *Tadmor* and *Kósa*

present the bending behavior of a trimorph beam bending structure with respect to the influence of the electromechanical coupling coefficient on the deviation moment of each individual layer [14]. *Smits* and *Choi* describe the electromechanical characteristic of heterogeneous bimorph bending structures subjected to different external mechanical and electrical quantities. Assuming thermodynamic equilibrium, they derive the universal deformation state equations for the tip of a bimorph by means of energetical considerations [15].

In the paper of *DeVoe* and *Pisano* a new model for the determination of the free tip deflection of piezoelectric multilayer beam bending actuators under the influence of an electrical load is presented. Thereby, the analytical formulation is based on a matrix equation combining mechanical, geometrical and elastic quantities of the individual layers [16]. *Meng*, *Mehregany* and *Deng* also consider piezoelectric bending actuators as multilayer system. Here, the analytical formulation of the beam bender's tip is in the foreground of discussion [17]. *Weinberg* extends the modeling of piezoelectric multilayer beam bending actuators taking the neutral axis position into account. The description of the deformation state with respect to external quantities such as force, moment and electrical voltage is restricted to the tip of the free bender [18].

All contributions mentioned above are characterized by extremely expanded analytical computations concerning the description of the static behavior of the interesting bending structure. The analytical calculations of deflection, bending angle, volume displacement and electrical charge effected by externally subjecting quantities are carried out partially and mostly for the bender's tip only. Based on this fact, the serious disadvantage arises, that the deformation state of the spatially expanded bending actuator can not be derived for arbitrary points over the entire length of the beam. Thus, important information is lost.

In order to disengage from special bending actuator types, the analytical formulation of a piezoelectric multilayer beam bending actuator consisting of n layers is pursued in the present book. Thereby, both passive elastic and active piezoelectric behavior is to be assigned each individual layer. The possibility of computing the deflection, bending angle, volume displacement and electrical charge effected by externally subjecting quantities such as force, bending moment, pressure load and electrical voltage for arbitrary points over the entire length of the beam shall be provided by an energetical description. The formulation of the equations describing the static behavior is to be kept in such a general form, allowing both users in industry and research organisations to derive the appropriate formulation for their given bender design.

Dynamics of Piezoelectric Beam Bending Actuators

Regarding the analytical description of the dynamic behavior of piezoelectric bending actuators such as a monomorph, bimorph, trimorph and also

a multilayer system, a large number of publications have been published in the scientific literature, too. In the following some representative works are summarized.

Smits and *Ballato* describe the dynamic behavior of a bimorph bending structure, which is excited to bending vibrations by external harmonic quantities such as forces, bending moments, pressure loads and electrical driving voltages [19]. The resulting harmonic quantities deflection, bending angle, volume displacement and generated charge are combined by a dynamic admittance matrix with the externally affecting loads. The dynamic behavior of a homogeneous bimorph structure caused by a harmonic driving voltage is described in a work by *Coughlin*, *Stamenovic* and *Smits* in order to determine the modulus of elasticity of different materials [20]. Thereby, the shifting of the resonance frequency is a measure for the modulus of elasticity being determined.

In a publication of *Smits*, *Choi* and *Ballato* the development of the first fundamental mode and its transition to the first antiresonance is discussed amongst the description of the resonance behavior of a bimorph bending structure [21]. The starting point for these considerations is the dynamic admittance matrix of a piezoelectric bimorph formulated by *Smits* and *Ballato* [19]. *Yao* and *Uchino* extend a dynamically driven bimorph with a flexible plate attached at the free bender's tip [22]. The resonance behavior of the realized structure is described as an electromechanical system with lumped parameters. Computational results based on the theoretical formulation are compared to measurement results. In the work of *Sitti*, *Campolo*, *Yan* and *Fearing* the dynamic behavior of a unimorph beam bending structure is discussed [23]. Thereby, the derivation of design criteria for the specific influence of the resonance behavior of piezoelectric two-layer systems is in the foreground of discussion.

On the base of Hamilton's principle *Fernandes* and *Pouget* formulate the system of differential equations describing the dynamics of a bimorph [24]. Thereby, a special attention is turned to the layer-wise modeling of the electrical potential. The publication of *Brissaud*, *Ledren* and *Gonnard* elaborates on the analytical formulation of the dynamic behavior of piezoelectric two-layer systems in addition to the description of their static behavior [10]. Thereby, the determination of the position-dependent fundamental mode and the associated resonant frequency are in the focus. *Seitz* and *Heinzl* present a microfluidic device based on piezoelectric bimorph structures [25]. The modeling of the dynamic behavior of the actuator structures takes place with the help of the simplified differential equation for flexural vibrations.

The description of the dynamics of a trimorph bending structure is the content of a publication of *Rogacheva*, *Chou* and *Chang* [26]. The computational results prognosticated from the developed theoretical formulations are compared to measurement data. *Kyu Ha* formulates the admittance matrix of bimorph bending structures derived by *Smits* and *Choi* [15] for a symmetrically layered trimorph [27, 28]. Based on Hamilton's variation principle, the

differential equation system describing the dynamics is derived. The derivation of the admittance matrix is based on matrix calculus.

Energetical principles within the context of Lagrange's formalism and application of Hamilton's variation principle are presented in several publications in order to determine the dynamics describing differential equation system of piezoelectric multilayer beam bending actuators [29–33]. All analytical formulations are based on the linear theory of piezoelectricity and contain the coupling of mechanical deformations with the charge equation of electrostatics.

All work inherent is the serious disadvantage, that the equivalent viscose damping caused by internal and external friction effects remains unconsidered in the dynamic formulations. This fact leads to inevitable singularities concerning flexural vibrations in the case of resonance, which does not correspond at all to the development of bending vibrations. Furthermore, it should be noted, that the differential equation for flexural vibrations is a function of the beam length coordinate and time. In most publications, the separation formalism concerning the location and time functions remains unconsidered. However, it is this formalism, that in connection with the characteristic equation for clamped-free beam bending actuators allows for the analytical description of the temporal and spatial development of the fundamental and higher order vibrational modes.

Therefore, in the present book, the describing differential equation system of a piezoelectric multilayer beam bending actuator consisting of n layers is derived for the first time, based on Hamilton's principle with respect to Rayleigh's dissipation function. In a next step, the analytical formulation of the spatial and temporal vibration response of piezoelectric multilayer actuators both, for the fundamental mode and for the modes of higher order follows. With the help of the general formulation, the engineer and scientist working in practice shall be given the possibility to prognosticate the dynamic behavior for any design of beam bending actuators. By variation of selected mechanical, geometrical and elastic parameters, the dynamic characteristic of any kind of piezoelectric bending actuators can be affected purposefully.

Equivalent Circuit Representation of Piezoelectric Beam Bending Actuators

The description of piezoelectric bending actuators as electromechanical systems within the context of the network theory has been also described in a large number of publications.

In a publication of *Ballato* the equivalent network representation of a piezoelectric two-layer system is worked out [34] based on the admittance matrix formulated by *Smits* and *Choi* [15]. Thereby, the description is concentrated

on the free tip of the bender. *Sherrit, Wiederick, Mukherjee* and *Sayer* develop an electromechanical circuit representation for a three-layered piezoelectric stack actuator concentrating on the low frequency range up to the first resonance [35]. A theoretical formulation of the dynamic behavior of a bimorph bending structure within the context of the network theory based on distributed parameters is the content of the publication by *Tilmans* [36]. However, the represented structure of the electromechanical network can only be applied for low frequencies. The electromechanical network representation of a trimorph bender in form of a five-port structure (ten-poles) is presented by *Cho, Pak, Han* and *Ha*. *Kyu Ha* describes an asymmetrical trimorph bending structure by means of an impedance matrix [28]. By means of this matrix, he derives a four-port network representation (eight-poles). Based on the dynamics describing differential equation system of a multilayer beam bending actuator [29], *Aoyagi* and *Tanaka* present the electrical network representation of piezoelectric multilayer bending actuators [37]. The admittances and resonant frequencies are calculated and compared with experimental results.

In all publications mentioned above the formulation of the electromechanical circuit representation is based on complex analytical equations. A clear representation of the mechanically interacting quantities such as masses, coefficients of friction and compliances and their interconnection is lost. Furthermore, the circuit representation is only formulated for the resonance of the fundamental mode. Modes of higher order remain unconsidered in their electromechanical representation. Therefore, in the present book the equivalent network representation of piezoelectric bending actuators consisting of n layers is developed step by step. Particularly, the description of the higher order modes within the context of the network theory is in the focus of discussion.

Bending Actuators with Integrated Sensors

Concerning the realization of piezoelectric actuators with integrated sensor technology, a large number of publications can be found [38–65]. The realized sensor concepts are based e.g. on the sensor characteristics of piezoceramic materials or the resistance change of thick film and metallic strain gage sensors. As a rule, the suggested solutions show one or more restrictions meaning that one of the demanded boundary conditions is not satisfied for the sensor completely integrated in the actuator. The solutions are characterized by the fact, that the sensor needs external components and is not able to detect the bender's tip deflection independently of external influences. A further restriction is the suitability for dynamic applications, however static and quasi-static applications are not possible [66].

The doctoral thesis of A. J. Schmid [67], written within the federal german research project EPIETEC mentioned above, describes the conceptual draw up, development and realization of an economical sensor for detecting forces and

deflections based on a capacitive strain gage. Hence, in this book a detailed and separate presentation is abandoned, instead a summarized overview will be given. On the one hand this book focuses on the realization of a sensor electronics for capacitive strain sensors and on the other on the development and realization of an inductive proximity sensor in combination with a high-accuracy electronic circuit for tip deflection measurements. Particularly, the inductive sensor principle offers the possibility of static, quasi-static and dynamic measurements.

1.4 Textual Focus of the Book

The present book is divided into the chapters described below.

Chapter 2 concentrates upon the phenomenological description of the direct and inverse piezoelectric effect. Thereby, the piezoelectric lead zirconate titanate system PZT is in the foreground of discussion. Its behavior in the low-level and high-level signal mode is described on the base of micro-physical domain and domain switching processes. Thus, the understanding of the macroscopic behavior of piezoelectric transducers is opened generally. With the transition to the standard forms of piezoelectric bending actuators the important terms of the bending structures used in practice are defined.

The description of the strain and stress state as well as the elastic behavior of a piezoelectric crystal structure follows in chapter 3. Their knowledge provides a basis for the derivation of the piezoelectric constitutive equations of state corresponding to the respective application. With the help of the constitutive equations, the electrical enthalpy by means of Legendre's transformation is deduced, being indispensable for the formulation of the dynamic behavior of piezoelectric bending actuators. The term of strain energy forms the fundamental of the structural dynamics consideration. The energy density of the elastic deformation and the energy density of the electric field are described in detail serving as starting point for all further considerations.

The formulation of the static behavior of n layered piezoelectric beam bending actuators in closed form analysis is the content of chapter 4. Thereby, Bernoulli's hypothesis of bending theory is the physical base. In combination with the basic laws of elastostatics, Bernoulli's hypothesis allows for the calculation of the neutral axis position. It is integrated into the general formulation of the flexural rigidity and the internal piezoelectric moment and represents one of the central quantities in all further considerations. With respect to the linear piezoelectric constitutive equations, energetical considerations lead to the total stored energy of a bending actuator in deformation state. With the principle of minimum total potential energy the combining equations between extensive affecting quantities such as force, bending moment, pressure load

and electrical driving voltage and the resulting intensive quantities like deflection, bending angle, volume displacement and electrical charge are derived for any point over the entire length of the actuator. The combining equations are presented in form of a coupling matrix.

Chapter 5 addresses the fundamentals concerning the description of the dynamics of piezoelectric bending actuators. At first, the important Lagrange formalism within the context of theoretical mechanics is developed considering dissipative forces, which extract energy from an oscillating system. Thereby, the internal and external frictional forces are integrated into Lagrange's formalism based on Rayleigh's dissipation function. In the next step, the transition to the modified Hamilton principle follows, which can be applied directly to the electrical enthalpy formulated in chapter 3. From Hamilton's principle the describing differential equations of a piezoelectric multilayer beam bending actuator are deduced in concentrated form.

In chapter 6 the differential equation system gained in chapter 5 is used for the description of the time-harmonic, dynamic behavior of piezoelectric multilayer actuators in closed form analysis. In analogy to the static description, the connection of the extensive dynamic quantities with the resulting intensive dynamic quantities is presented for arbitrary points over the entire length of the actuator by means of a dynamic admittance matrix.

The systematic development of a general circuit representation of piezoelectric multilayer beam bending actuators within the context of the network theory is the content of chapter 7. Particularly, the frictional forces arising with bending vibrations are considered in the sense of the equivalent viscous damping. Considering the differential bending behavior of an electrically driven multilayered actuator segment, the general representation of the electromechanical coupling of electrical and mechanical quantities will follow by means of a gyratoric two-port (four-pole) network. The combination of the two-port network with the translatory and rotatory quantities coupling four-port network leads to the general representation of a piezoelectric multilayer beam bending actuator as an electromechanical system with respect to the development of higher order modes.

A special measurement setup for piezoelectric beam benders as well as the experimental characterization of the static and dynamic behavior of a realized monomorph beam bending actuator in multilayer technology form the main focus of chapter 8 and 9. Amongst the the functionality of the measurement setup, the individual measurement tasks and their experimental realization are discussed in detail. The measurement setup allows for the experimental validation of the derived theoretical models. In order to verify the theoretical formulations, analytical calculations are compared to measurement data.

In chapter 10 the transition to the second main focus of the present book follows. Thereby, the development and realization of a smart sensor-actuator-system in the form of a pneumatic micro valve with integrated deflection sensor are in the focus of discussion. The requirements for the integrated deflection sensor are presented and serve as a selection base of favoured sensor concepts.

In order to detect the beam bender's tip deflection, the conceptual draw up, development and realization of a sensor electronics for capacitive strain sensors have been occurred within the research project EPIETEC. In chapter 11 the working principle of a capacitive strain sensor for tip deflection measurement is discussed. Furthermore, the sensor electronics and its structure are discussed in detail. The characterization of the realized sensor-actuator-system in combination with the developed sensor electronics follows.

Besides the realization of a sensor electronics for capacitive strain sensors, additionally, a second deflection sensor based on an inductive proximity principle has been developed. The inductive proximity sensor, which is based on eddy current effects, offers the possibility of sensor integration in smallest spaces in combination with a high-accuracy electronic circuit. In chapter 12 the essential structure and the operational mode of the inductive proximity sensor are discussed. In order to determine analytically the relative change of inductance as a function of the beam bender's tip deflection, an electrodynamic approach is formulated verifying experimental measurement results in an outstanding way. By theoretical formulation, a deep physical understanding of the mechanism of the inductive proximity sensor based on eddy current effects is given simultaneously. In a next step, the characterization of the inductive proximity sensor concerning its static and dynamic transient behaviour is in the foreground of discussion.

Chapter 13 summarizes the results of the present book. An extension of the formulated structure dynamical aspects is presented. Furthermore, new application possibilities for smart piezoelectric multilayer beam bending actuators are outlined. An outlook on continuing scientific works and technical application fields conclude this book.

Theoretical Aspects and Closed Form Analysis

Piezoelectric Materials

In this chapter, it is dwelled on the discovery of piezoelectricity and the formation of the direct and the inverse piezoelectric effect is described qualitatively. The development of the technical application of piezoelectric materials shows, how various their application fields are. Especially, the attention is given to the technical importance of the piezoelectric lead zirconate titanate system PZT. For a better understanding of its behavior in the low-level and high-level signal mode, it is necessary to dwell on the domain and domain switching processes. The electromechanical behavior of the piezoelectric ceramic system PZT provides the realization of piezoelectric beam bending actuators, which are of highest importance in many application fields piezoelectric actuators and sensors are applied.

2.1 Discovery of Piezoelectricity

In the year 1880, the piezoelectricity was discovered by the brothers Pierre and Jacques Curie [68]. However, this did not happen randomly. A long time ago, in India and Ceylon the mysterious behavior of tourmalins was already well-known. If tourmalins were put into hot ash, at one side they attracted ash particles, at the opposite side they were rejected. After some time the effect of attraction and rejection inverted. At the beginning of the 18th century, traders brought the tourmalin crystals to Europe. 1747 Linné called the tourmalin crystals *lapis electricus*. In the following century, some researchers endeavored to find a connection between the mechanical pressure effect and the electricity. Becquerel was aware of the fact, that such an effect could be particularly expected with crystals. In the year 1877, Lord Kelvin established the correlation between pyroelectricity and piezoelectricity. It could be verified, that the most part of the pyroelectric charge of the tourmalin is ascribed to the formation of piezoelectric surface charge caused by the elastic crystal deformation under temperature changes. First, the brothers Pierre and Jacques